KVANTRÖV 1

Anders

I 5 Rutherfords spridningsexperiment

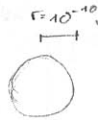
Thomson

$E_k = 5{,}0\,\text{MeV}$ $\alpha(He^{2}) \longrightarrow \overset{\circ}{A}u^{79}$ $\quad E_{pot} = \dfrac{Q_1 Q_2}{4\pi \varepsilon_0 r}$

När är $E_k = E_{pot}$? $\quad E_k = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r} \Leftrightarrow r = \dfrac{Q_1 Q_2}{4\pi \varepsilon_0 E_k}$

Sätt in $Q_1 = Q(He^2) = 2e$

$Q_2 = Q(Au^{79}) = 79e$ $\quad e = 1{,}6\cdot 10^{-19}\,C$



$\varepsilon_0 = 8{,}85\cdot 10^{-12}\, C/Vm$

$E_k = 5\cdot 10^6\, e\cdot 1\,V$

$$\Rightarrow r = \frac{2\cdot 79\, e^2}{4\pi\cdot 8{,}85\cdot 10^{-12}\cdot 5\cdot 10^6\, e\, V\, C/Vm} =$$

$$= \frac{2\cdot 79\, e}{4\pi\cdot 8{,}85\cdot 10^{-12}\cdot 5\cdot 10^6}\,\frac{m}{C} = \;= 4{,}5\cdot 10^{-14}\, m$$

II 6

$$\nu = \frac{c}{\lambda} \Rightarrow d\nu = -\frac{c}{\lambda^2}d\lambda$$

$$E = \int_{\nu_0}^{\nu_1} u_\nu\, d\nu = -\int_{\lambda_1}^{\lambda_0} u_\nu \frac{c}{\lambda^2}\, d\lambda \quad \text{och} \quad E = \int_{\lambda_0}^{\lambda_1} u_\lambda\, d\lambda = -\int_{\lambda_1}^{\lambda_0} u_\lambda\, d\lambda$$

$$\therefore\ u_\lambda\, d\lambda = u_\nu \frac{c}{\lambda^2}\, d\lambda \Leftrightarrow u_\lambda = u_\nu\frac{c}{\lambda^2}$$

Plancks strålningslag

II 7

$$u_\nu(\nu,T) = \frac{8\pi h\nu^3}{c^3}\left(e^{h\nu/k_BT}-1\right)^{-1}$$

$$\Rightarrow \frac{\partial u_\nu}{\partial \nu} = 3\frac{8\pi h\nu^2}{c^3}\left(e^{h\nu/k_BT}-1\right)^{-1} - \frac{8\pi h\nu^3}{c^3}\left(e^{h\nu/k_BT}-1\right)^{-2}\frac{h}{k_BT}e^{h\nu/k_BT} =$$

$$= \frac{8\pi h\nu^2}{c^3}\left(e^{h\nu/k_BT}-1\right)^{-1}\left[3 - \frac{h\nu}{k_BT}\left(e^{h\nu/k_BT}-1\right)^{-1}e^{h\nu/k_BT}\right].$$

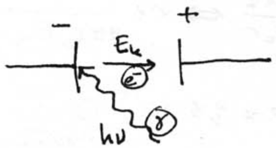
$$\ldots \Rightarrow \frac{\partial u_\nu}{\partial \nu} = 0 \Rightarrow (\text{numeriskt}):\ \frac{h\nu}{k_BT} = 2{,}8$$

$$\boxed{\left(3 - \frac{xe^x}{e^x-1}\right) = 0}$$

II 12 Fotoelektrisk effekt

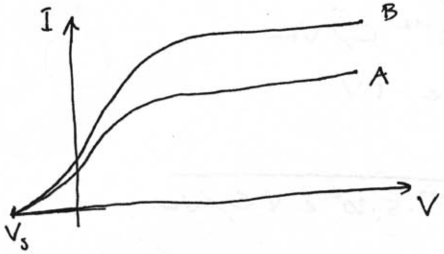
Utträdesarbetet W krävs för att slå loss elektron

$\Rightarrow \boxed{E_{k,max} = h\nu - W}$ ← Det som blir över används till E_k (Vissa elektroner är hårdare bundna)



Strypspänning, V_s : $\boxed{eV_s = h\nu - W}$

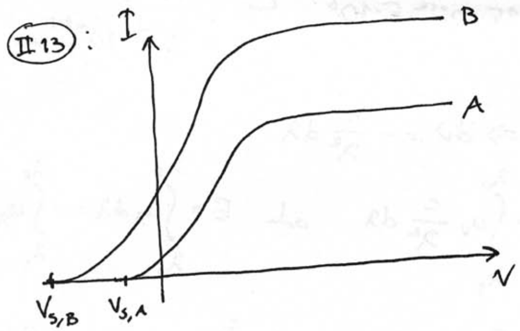
Lika mycket eV bakåt som $E_{k,max}$



Samma strypspänning, men olika resulterande intensitet (fotoström)

∴ Olika intensitet hos fotonerna

II 13 :



Olika strypspänning

∴ olika W (olika material)

~~och~~/eller

olika $h\nu$ (olika ljusfrekvens)

II 17

$W_{Na} = 2{,}0\,eV$ $\quad \lambda = 4000\,Å$ $\quad \left(E_\gamma = \dfrac{hc}{\lambda}\right)$ $\quad E_k$?

$$E_{k,max} = E_\gamma - W_{Na} = \frac{hc}{\lambda} - 2{,}0\,eV = \frac{6{,}6\cdot 10^{-34}\cdot 3\cdot 10^8}{4000\cdot 10^{10}}\,\frac{J\,s\,m/s}{m} -$$

$$- 2\cdot 1{,}6\cdot 10^{-19}\,J = -1{,}75\cdot 10^{-19}\,J$$

...

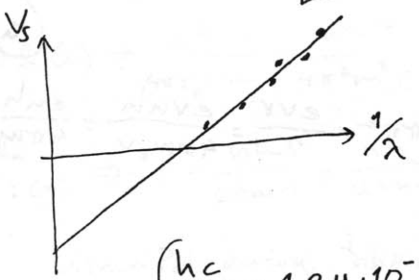
$[h = 6{,}626\cdot 10^{-34}\,Js]$

... Hastighet? Icke-rel: $E = \frac{mv^2}{2}$

$\Rightarrow v = \sqrt{\frac{2E_k}{m_e}} = 620\,000$ m/s (<<c OK!)

→ II 20 Kurvanpassning...

$$eV_s = h\nu - W \Leftrightarrow \boxed{V_s = \frac{hc}{e}\frac{1}{\lambda} - \frac{W}{e}}$$



Plotta $1/\lambda$ mot V_s

: Mathlab/Mathematica/..

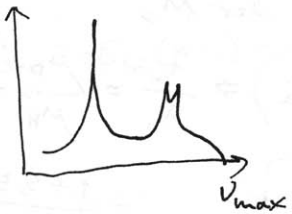
$\Rightarrow V_s = 1{,}24 \cdot 10^{-6}\frac{1}{\lambda} - 2{,}27$

$$\therefore \begin{cases} \frac{hc}{e} = 1{,}24 \cdot 10^{-6} \ \frac{Jm}{C} \\ \cancel{\ } = 2{,}27 \text{ V} = \frac{W}{e} \end{cases}$$

$\Rightarrow$ a) $W = 3{,}64 \cdot 10^{-19}$ J $= 2{,}27$ eV

b) $h = 6{,}62 \cdot 10^{-34}$ Js

II 23 Kontinuerlig röntgenstrålning



$h\nu_{max} = eV$ slungar elektroner från katod till anod

(dvs all energi från elektronen (eV) till en foton ($h\nu$))

$V = 59{,}9 \cdot 10^3$ V $\quad \lambda_{min} = 20{,}6 \cdot 10^{-12}$ m

$$h\frac{c}{\lambda_{min}} = eV \Leftrightarrow h = \frac{eV\lambda_{min}}{c} = \frac{1{,}602 \cdot 10^{-19} \cdot 59{,}9 \cdot 10^3 \cdot 20{,}6 \cdot 10^{-12}}{3 \cdot 10^8} \ \frac{Jm}{m/s} =$$

$$= 6{,}59 \cdot 10^{-34} \text{ Js}$$

II 35

$m = iA$, $\quad i = \frac{q}{t} = \frac{e}{2\pi r / v}$ ← ström

$A = \pi r^2$

Bohr-Sommerfelds kvantiseringsregel

$$\int_0^{2\pi} L\, d\phi = nh \Leftrightarrow n = \frac{2\pi L}{h} \quad \left(\begin{matrix} L = \text{rörelsemängdsmoment} \\ = mvr \end{matrix} \right)$$

$$\Rightarrow r = \frac{nh}{2\pi m v}$$

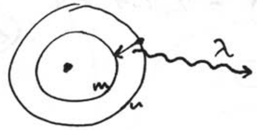
$\hbar \equiv \frac{h}{2\pi}$

$$\therefore m = iA = \frac{eV}{2\pi r}\pi r^2 = \frac{evr}{2} = \frac{evnh}{4\pi m_e v} = \frac{enh}{4\pi m_e} \left(= \frac{en\hbar}{2m_e} \right)$$

→ II 38 Spektrallinjer

$\lambda_1 = 486{,}1320$ nm, $\lambda_2 = 485{,}9975$ nm

Om $\lambda_1 = \lambda_H$, kan då $\lambda_2 = \lambda_D$?



Rydbergs formel: Härledd; kamp. s 19

$$\boxed{\frac{1}{\lambda} = Z^2 R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)}$$

obs! Oändlig kärnmassa ⇒ $\mu = m_e$

Allmänt: $R = \frac{\mu e^4}{8\varepsilon_0^2 c h^3}$, μ = reducerad massa $= \frac{m_1 m_2}{m_1 + m_2} = \frac{m_1}{m_1/m_2 + 1}$

Här: $m_1 = m_e$, $m_2 = M_H$ eller M_D

$$\Rightarrow \frac{1}{\lambda} = Z^2 \frac{\mu e^4}{8\varepsilon_0^2 c h^3}\left(\frac{1}{m^2} - \frac{1}{n^2} \right) \Rightarrow \frac{\lambda_H}{\lambda_D} = \frac{\mu_D}{\mu_H} = \frac{m_e/M_H + 1}{m_e/M_D + 1} =$$

$$\left[\begin{matrix} \frac{m_e}{M_H} \approx \frac{m_e}{m_p} = \frac{9{,}11 \cdot 10^{-31}}{1{,}67 \cdot 10^{-27}} = 5{,}46 \cdot 10^{-4} \\ \frac{m_e}{M_D} \approx \frac{m_e}{2m_p} = 2{,}73 \cdot 10^{-4} \end{matrix} \right]$$

$$= \frac{1 + 5{,}5 \cdot 10^{-4}}{1 + 2{,}7 \cdot 10^{-4}} = 1{,}00028$$

och

$$\frac{\lambda_1}{\lambda_2} = \frac{486{,}1320}{485{,}9975} = 1{,}00028$$

II 40   Joniseringsenergi för positronium

Två kraftuttryck: $(F) = \dfrac{e^2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r} = (ma)$

$\uparrow$ $F_{Coulomb}$ $\uparrow$ $F_{centripetal}$

$\Leftrightarrow v^2 = \dfrac{e^2}{4\pi\varepsilon_0 r m}$ (1)

Sedan tidigare: $nh = 2\pi m v r \Leftrightarrow v = \dfrac{nh}{2\pi m r}$ (2)

$$(1) \,\&\, (2) \Rightarrow \frac{e^2}{4\pi\varepsilon_0 r m} = \frac{n^2h^2}{4\pi^2 m^2 r^2} \Leftrightarrow r = \frac{n^2h^2\varepsilon_0}{\pi m e^2}$$

Detta r in i (2) $\Rightarrow v = \dfrac{nh}{2\pi m r} = \dfrac{nh}{2\pi m} \cdot \dfrac{\pi m e^2}{n^2h^2\varepsilon_0} = \dfrac{e^2}{2nh\varepsilon_0}$

Kin. + Pot. energi blir:

$$E_k + E_v = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{1}{2}m\frac{e^4}{4n^2h^2\varepsilon_0^2} - \frac{e^2}{4\pi\varepsilon_0}\frac{\pi m e^2}{n^2h^2\varepsilon_0} =$$

$$= \frac{1}{8}m\frac{e^4}{n^2h^2\varepsilon_0^2} - \frac{1}{4}m\frac{e^4}{n^2h^2\varepsilon_0^2} = -\frac{1}{8}m\frac{e^4}{n^2h^2\varepsilon_0^2}$$

Vilken massa? Reducerad massa! $m_1 = m_2 = m_e$

$$\Rightarrow \mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2} \Rightarrow E_{tot} = -\frac{1}{16}m_e\frac{e^4}{n^2h^2\varepsilon_0^2}$$

$$E_{JON} = -E_{tot}(n=1) = \frac{m_e e^4}{16h^2\varepsilon_0^2} = \left\{ \begin{array}{ll} m_e = 9{,}11\cdot 10^{-31} & h = 6{,}6\cdot 10^{-34} \\ e = 1{,}602\cdot 10^{-19} & \varepsilon_0 = 8{,}85\cdot 10^{-12} \end{array} \right\}$$

$$= \underline{\underline{6{,}9\ \text{eV}}}$$