Hugo Strand

Kvantföv. 8

7/10-08, FL 72

13^{15}–15^{00}

$\underline{\text{XII}}$. 7, 8, 9, 10

$\underline{\text{XIII}}$. 2, 4, 7

XII.7/ Natrium atomen $\approx$ en el. atom

$$\hat{H} = \underbrace{\hat{H}_0}_{\text{partícentral fält Ham.}} + \overbrace{\frac{\xi(r)}{\hbar^2}\hat{L}\cdot\hat{S}}^{\hat{H}_{SB}}$$

Spinn-Bankoppling ger finstruktur i spektra!

$3p \rightarrow 3s$ överg delas i två komp.

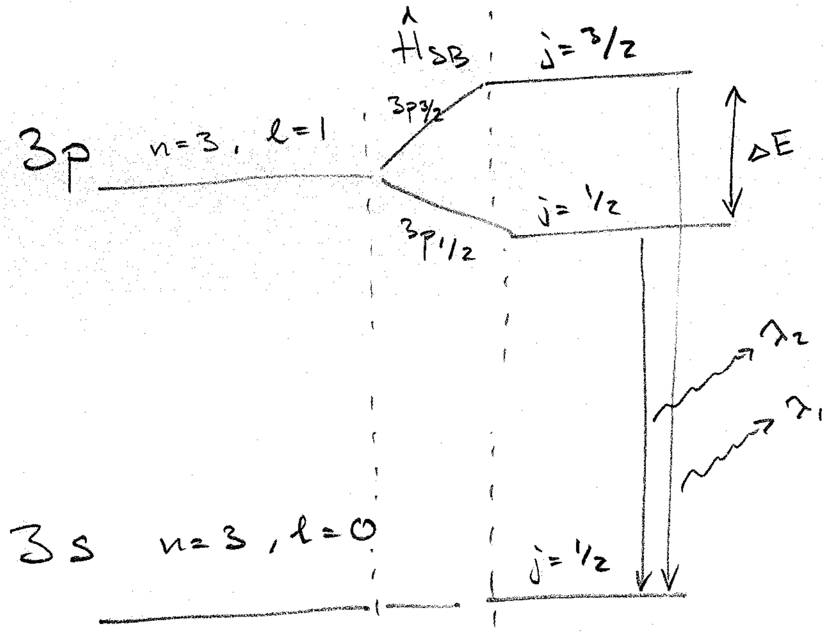
Gula D-dubbletten : $\lambda_1 = 5889.953$ Å
$\lambda_2 = 5895{,}923$ Å

Ber väntevärdet av $\xi(r)$ (spinnbank. funk.) i 3p-tillst. i eV $\Rightarrow \langle 3p|\xi(r)|3p\rangle = \underline{\xi_{31}}$

Spinn ban koppl: Goda kvanttal $n\,l\,j\,m_j$

$j = l \pm 1/2$ för en el. $\quad E_{nlj} = E^{(0)}_{nl} + \frac{\hbar^2}{2}[j(j+1) - l(l+1) - s(s+1)] \times \xi_{nl}$

överg. regler:
$\Delta l = \pm 1$
$\Delta j = 0, \pm 1$
Ok i dettafall



ej $j = -\frac{1}{2}$ ty längd på tot. magn mom. (12.34 komp)

$$\Delta E = E_{\lambda_1} - E_{\lambda_2} = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = E_{31\frac{3}{2}} - E_{31\frac{1}{2}} =$$

$$= \frac{\hbar^2}{2}\xi_{31}\left[\frac{3}{2}\left(1+\frac{3}{2}\right) - \frac{1}{2}\left(1+\frac{1}{2}\right)\right] = \frac{3\hbar^2}{2}\xi_{31}$$

$$\frac{3\cdot 5}{4} - \frac{3}{4} = \frac{12}{4} = 3$$

$$\hbar^2\xi_{31} = \frac{2}{3}hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \approx 1{,}4\,\text{meV}$$ energi skala för SB koppl.

XII. 8/ Lägg på magnetfält! $\mathbf{B} = B\hat{z}$

Ange hur natriumdubbletten uppsplittras:

Nu: $$\hat{H} = \hat{H}_0 + \underbrace{\frac{\xi(r)}{\hbar^2}\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}}_{\hat{H}_{SB}} + \underbrace{\frac{\mu_B \mathbf{B}}{\hbar}(\hat{L}_z + 2\hat{S}_z)}_{\text{Zeeman eff.}}$$

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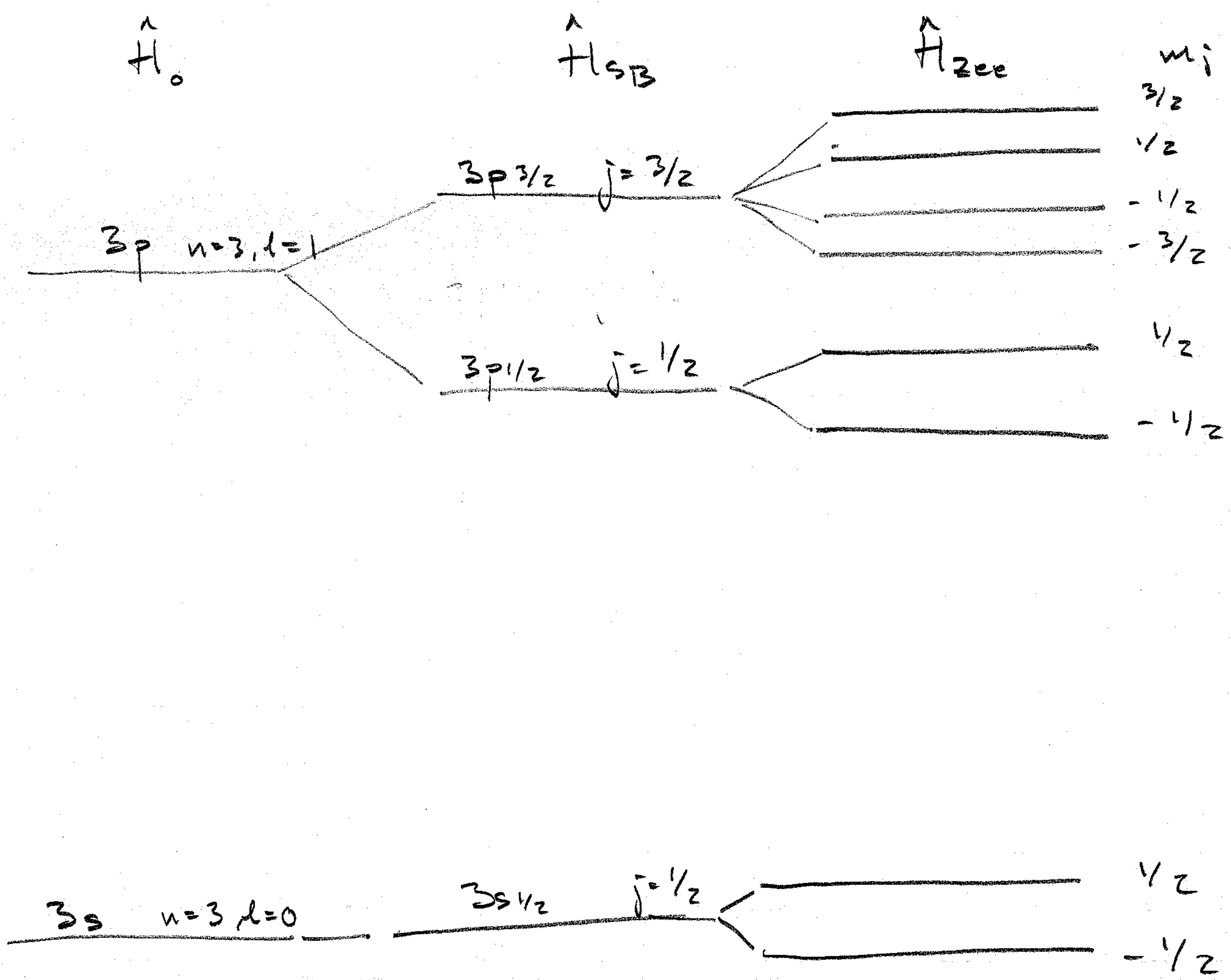
a) Energier: $E_{nljm_j} = E_{nlj} + g_j \mu_B B m_j$

g_j Landé's g-faktor: $g_j = 1 + \dfrac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$

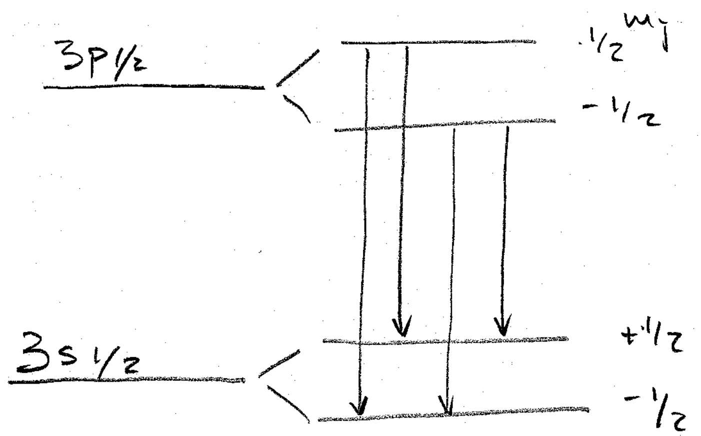
$j = l \pm 1/2$; $m_j = -j, -j+1, \dots, j-1, j$



XII. 8 forts / Tillåtna överg.

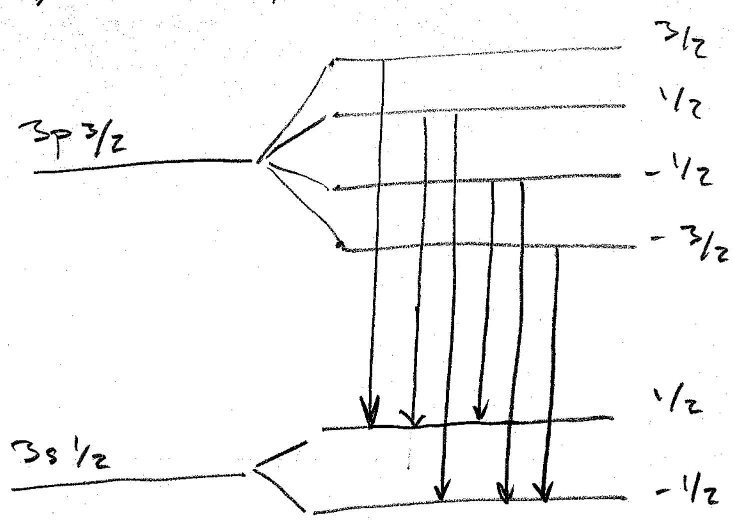
$$\begin{cases} \Delta l = \pm 1 \\ \Delta j = 0, \pm 1 \\ \Delta m_j = 0, \pm 1 \end{cases}$$

Första linjen i Dubletten:



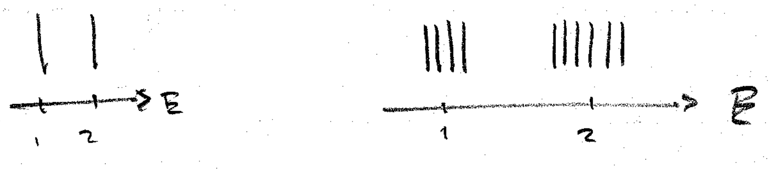
$\Rightarrow$ $3p_{1/2} \to 3s_{1/2}$ splittras i 4 delar

Andra linjen i Dubletten



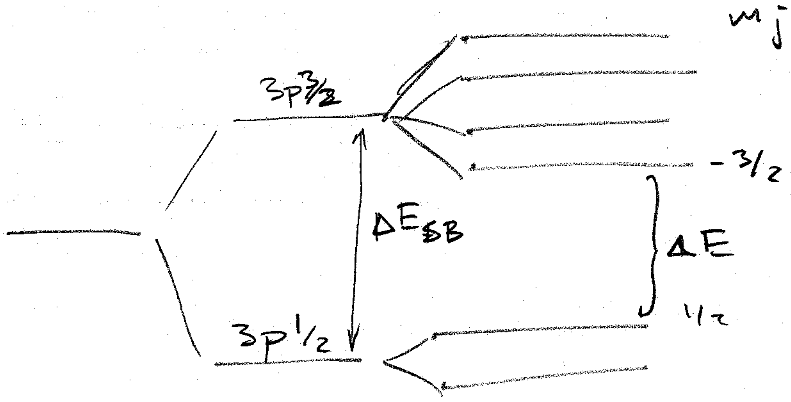
$\Rightarrow$ $3p_{3/2} \to 3s_{1/2}$ splittras i 6 delar

Dvs. 5896Å delas i 4 & 5890Å i 6 delar



XII.8/ b) För vilket magn fält B sammanf.

Lägsta $3p_{3/2}$ niv m. högsta $3p_{1/2}$



Sammanf. => $\Delta E = 0$

$$\Delta E = \Delta E_{SB} + g_{3/2}\mu_B B\left(-\frac{3}{2}\right) - g_{1/2}\mu_B B\left(\frac{1}{2}\right)$$

$$g_{3/2} = \ldots = \frac{4}{3} \quad ; \quad g_{1/2} = \ldots = \frac{2}{3}$$

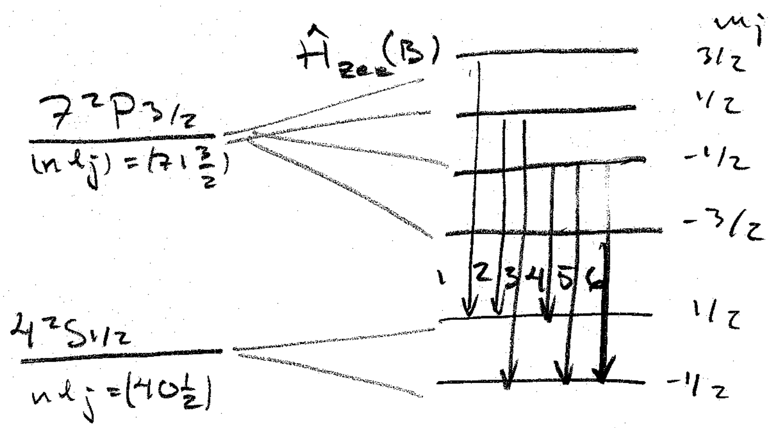
$$\Rightarrow \quad \Delta E = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) - \mu_B B \underbrace{\left(\frac{4}{3}\frac{3}{2} + \frac{2}{3}\frac{1}{2}\right)}_{7/3} = 0$$

$$\Rightarrow \quad B = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\frac{3}{7\mu_B} \approx \underline{\underline{15{,}8\ T}}$$

XII.9/ Kaliums P-serie har en spektrallinje motsv. överg. $7\,^2P_{3/2} - 4\,^2S_{1/2}$ m. $\lambda = 3217$ Å.

Landé's g-fakt. är $\frac{4}{3}$ för $P_{3/2}$ & 2 för $S_{1/2}$

Ber uppspaltningen av linjen i ett yttre magnetfält. $B = 2.14\,T$



Söker energiskillnaden ΔE mellan $(B=0)$ $7\,^2P_{3/2} \to 4\,^2S_{1/2}$ överg & $1, \dots, 6$ uppsplittrade överg.

$$\Delta E = \delta E_{3/2\,m_j} - \delta E_{1/2\,m_j}$$

$$\delta E_{j m_j} = g_j \mu_B B m_j \implies \frac{\delta E}{\mu_B B} = g_j m_j$$

	m_j	g_j	$\frac{\delta E_{j m_j}}{\mu_B B}$
$j = 3/2$	$3/2$	$4/3$	2
	$1/2$	|	$2/3$
	$-1/2$	|	$-2/3$
	$-3/2$	|	-2
$j = 1/2$	$1/2$	2	1
	$-1/2$	|	-1

$\implies$

$$\frac{\Delta E_1}{\mu_B B} = 2 - 1 = 1$$

$$\frac{\Delta E_2}{\mu_B B} = \frac{2}{3} - 1 = -\frac{1}{3}$$

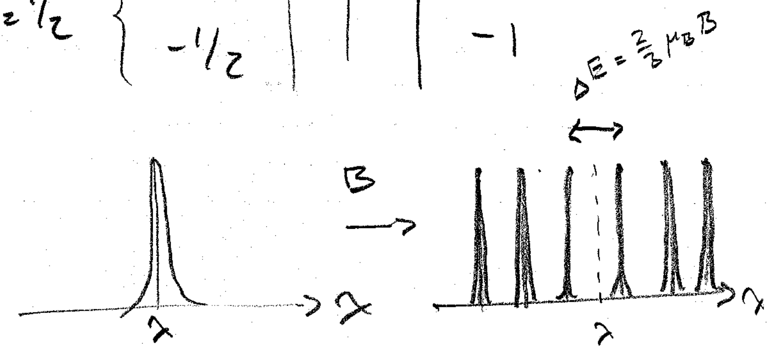
$$\frac{\Delta E_3}{\mu_B B} = \frac{2}{3} + 1 = \frac{5}{3}$$

$$\frac{\Delta E_4}{\mu_B B} = -\frac{2}{3} - 1 = -\frac{5}{3}$$

$$\frac{\Delta E_5}{\mu_B B} = -\frac{2}{3} + 1 = -\frac{1}{3}$$

$$\frac{\Delta E_6}{\mu_B B} = -2 + 1 = -1$$

$\implies$



XII.10/ Positronium. $S_{e^-} = S_{e^+} = \frac{1}{2}$

$$H = H_0 + \underbrace{\frac{A}{\hbar^2}\hat{S}_1 \cdot \hat{S}_2}_{\text{Koppling mellan spinnen hos } e^- \,\&\, e^+}$$

Grundtillst splittras i ${}^1S$ & ${}^3S$

S_{tot} är totala spinnet.

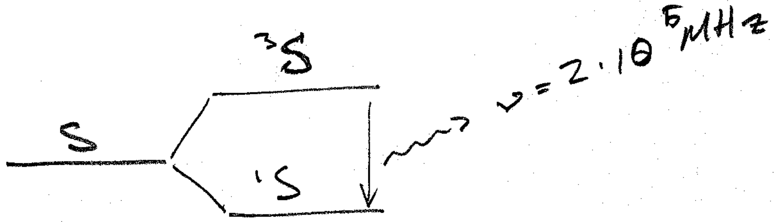
De två tillst. motsv.

$1 = 2S_{tot}+1$, $\underline{S_{tot}=0}$, $\ell = 0$

$\Rightarrow 3 = 2S_{tot}+1$, $\underline{S_{tot}=1}$

$S_{tot} = 0$: $\uparrow\downarrow$

$S_{tot} = 1$: $\uparrow\uparrow$



Ber: A

$$\Delta E = h\nu = \left\langle \frac{A}{\hbar^2}\hat{S}_1\cdot\hat{S}_2 \right\rangle_{{}^3S} - \left\langle \frac{A}{\hbar^2}\hat{S}_1\cdot\hat{S}_2 \right\rangle_{{}^1S}$$

$$\langle \hat{S}_1\cdot\hat{S}_2 \rangle = /\text{omskrivn.}/ = \left\langle \frac{1}{2}\left[(\hat{S}_1+\hat{S}_2)^2 - \hat{S}_1^2 - \hat{S}_2^2 \right] \right\rangle$$

$\hat{S}_1^2$ & $\hat{S}_2^2$ har egenv. $\langle \hat{S}_i^2 \rangle = \hbar^2 s_i(s_i+1) = \frac{3\hbar^2}{4}$

(samma för båda tillst.) $\qquad \underline{s_i = \frac{1}{2}} = S_{e^-} = S_{e^+}$

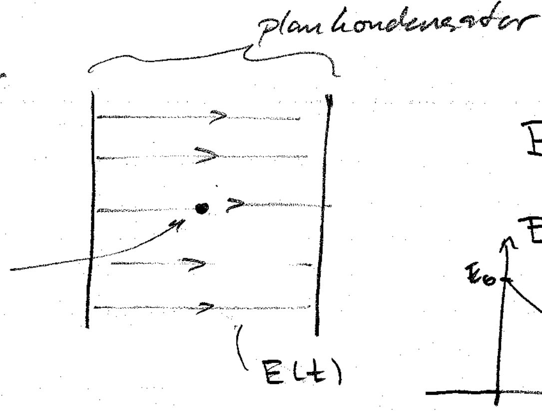
$$\langle (\hat{S}_1+\hat{S}_2)^2 \rangle = \langle \hat{S}_{tot}^2 \rangle = \hbar^2 S_{tot}(S_{tot}+1) = \begin{cases} 0, & {}^1S \\ 2\hbar^2, & {}^3S \end{cases}$$

$$\Rightarrow \Delta E = h\nu = \frac{A}{\hbar^2}\frac{1}{2}(2\hbar^2 - 0) = A\,!$$

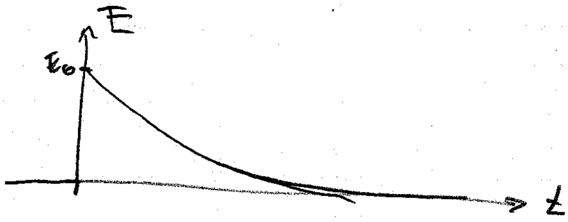
$$\underline{A = h\nu \approx 0{,}8 \text{ meV}}$$

XIII. 2

Väteatom i g.s. 1s $t=0$



$$E(t) = \begin{cases} 0, & t<0 \\ E_0 e^{-t/\tau}, & t\geq 0 \end{cases}$$



Ber: $P(2s)$ för $t\to\infty$ m. lägsta ordn störn teori.

$$H = H_0 + \underbrace{\hat{V}(t)}_{\text{liten störn.}} \quad ; \quad H_0 \psi_n = E_n \psi_n$$

$\Rightarrow$ Generell lösn: $\psi(t) = \sum_{n=0}^{\infty} c_n(t)\,\psi_n(r)\,e^{-i/\hbar E_n t}$

Vet $t=0$: $\psi = \psi_{100} \Rightarrow |c_{100}|^2 = 1$ låt $c_s = c_{100}$

Fermis gyllene regel:

$$c_k(t) = \delta_{ks} - \frac{i}{\hbar}\int_0^t \langle k|V(t')|s\rangle e^{i\omega_{ks}t'}dt'$$

$$\omega_{ks} = \frac{1}{\hbar}(E_k - E_s)$$

Nu $s=100$, $k=200$ (2s)

$\hat{V}(t)$? $\quad \mathbf{E} = E\hat{z}$; $\quad \mathbf{F} = e\mathbf{E}\hat{z} = -\nabla\hat{V}$

↑ verkande på el.

$$\Rightarrow \hat{V} = -\hat{z}\int \underset{\uparrow \text{konst.}}{F_z}\, dz = \underline{\underline{-eE(t)\,z\,\hat{z}}}$$

OBS! $l=0$ $\Rightarrow$ inget θ,φ ber.

$$\langle 200|\hat{V}(t)|100\rangle = \int dV\, \psi^*_{200}(r)\, eE(t)\underbrace{z}_{r\cos\theta}\psi_{100}(r) =$$

$$= eE(t)\int_0^{2\pi} d\varphi \underbrace{\int_0^{\pi}\cos\theta\sin\theta\, d\theta}\int_0^{\infty}\psi^*_{200}(r)\psi_{100}(r)\, r\, r^2 dr$$

$$\underbrace{\int_0^{\pi}\frac{1}{2}\sin 2\theta\, d\theta}_{\text{Hel period!}} = 0 \Rightarrow |c_{200}(t)|^2 = 0 = P(2s)_{t\to\infty}$$

E-fältet kan inte excitera atomen!

Eftersom fältet är homogent. Jmf Stern Gerlach

XIII. 4 / Gylleneregeln: Sannorlikheten för dipolöverg. best. av $\langle k | e\mathbf{p} | s \rangle$

Visa att: $\langle k | e\mathbf{p} | s \rangle = i\mu\omega_{ks} \underbrace{\langle k | e\mathbf{r} | s \rangle}_{\text{elektriskt dipolmom}}$

Lösn: $[\hat{H}_0, \mathbf{r}] = \left[\frac{\mathbf{p}^2}{2\mu} + V(\mathbf{r}), \mathbf{r}\right] =$

$= \frac{1}{2\mu}[\mathbf{p}^2, \mathbf{r}] = \frac{1}{2\mu}[p_x^2 + p_y^2 + p_z^2, x\hat{x} + y\hat{y} + z\hat{z}] =$ ↑ enhetsvekt.

$= \frac{1}{2\mu}\left([p_x^2, x]\hat{x} + [p_y^2, y]\hat{y} + [p_z^2, z]\hat{z}\right) =$

$= \Big/ \; [p_x^2, x] = p_x \underbrace{[p_x, x]}_{-i\hbar} + [p_x, x]p_x = -2i\hbar p_x \Big/ =$

$= \frac{-2i\hbar}{2\mu}(p_x\hat{x} + p_y\hat{y} + p_z\hat{z}) = -\frac{i\hbar}{\mu}\mathbf{p}$

$\Rightarrow \quad \underline{\mathbf{p} = \frac{i\mu}{\hbar}[H_0, \mathbf{r}]}$

Anv. $\langle k | e\mathbf{p} | s \rangle = \frac{i\mu e}{\hbar}\langle k | [H_0, \mathbf{r}] | s \rangle =$

$= \frac{i\mu e}{\hbar}\left(\underbrace{\langle k | H_0}_{= E_k\langle k|} \mathbf{r} | s \rangle - \langle k | \mathbf{r} \underbrace{H_0 | s \rangle}_{= E_s | s \rangle}\right) =$

$= \frac{i\mu e}{\hbar}\left(E_k \langle k | \mathbf{r} | s \rangle - E_s \langle k | \mathbf{r} | s \rangle\right) =$

$= i\mu \cdot \underbrace{\frac{E_k - E_s}{\hbar}}_{= \omega_{ks}} \langle k | e\mathbf{r} | s \rangle = i\mu\omega_{ks} \langle k | e\mathbf{r} | s \rangle$

Ok!

XII . 7 / 1D Harm osc. med massa m vinkelfrekv. ω & laddn q i elektriskt fält: $E = E_0 \cos \nu t$

Ber: överg. sannolikheten för överg.

a) $n: 0 \to 1$ b) $n: 1 \to 2$ c) $n: 0 \to 2$

Fermi:

$$c_{i \to f}(t) = -\frac{i}{\hbar} \int_0^t \langle f | V(t') | i \rangle e^{i \omega_{fi} t'} dt'$$

initial & final state

$$P(i \to f) = |c_{i \to f}|^2$$

Ber storm. V:

$E = E_0 \cos \nu t \Rightarrow F = q E_0 \cos \nu t$, $F = -\nabla V = -\frac{\partial}{\partial x} V$

$$V = -q E_0 x \cos \nu t$$

Tillstånds vågfunk:

$(\alpha = \sqrt{\frac{m\omega}{\hbar}})$

jämn $\psi_0 = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2} e^{-\frac{1}{2}\alpha^2 x^2}$

udda $\psi_1 = \left(\frac{\alpha}{2\sqrt{\pi}}\right)^{1/2} 2\alpha x e^{-\frac{1}{2}\alpha^2 x^2}$

jämn. $\psi_2 = \left(\frac{\alpha}{8\sqrt{\pi}}\right)^{1/2} (4\alpha^2 x^2 - 2) e^{-\frac{1}{2}\alpha^2 x^2}$

Vilka matris el. kommer vara $\neq 0$?

$$\langle f | V(t') | i \rangle = \int_{-\infty}^{\infty} \psi_f^* V(t') \psi_i \, dx = -q E_0 \cos \nu t \cdot \underbrace{\int_{-\infty}^{\infty} \psi_f^* \, x \, \psi_i \, dx}_{\text{om udda } = 0!}$$

a) $\langle \underset{u}{1} | \underset{u}{V} | \underset{j}{0} \rangle \neq 0 \Rightarrow |c_{0 \to 1}|^2 \neq 0$

b) $\langle \underset{j}{2} | \underset{u}{V} | \underset{u}{1} \rangle \neq 0 \Rightarrow |c_{1 \to 2}|^2 \neq 0$

c) $\langle \underset{j}{2} | \underset{u}{V} | \underset{j}{0} \rangle = 0 \Rightarrow |c_{0 \to 2}|^2 = 0 !$