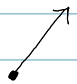
Föreläsning 24/9-13

$\mathbb{F} = -\nabla \phi$

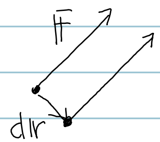
vektorfält — skalärt fält - "potential"

- Vad är kriteriet för att ett fält $\mathbb{F}$ ska ha en skalär potential

Mekaniktolkning



$\mathbb{F}(\mathbb{r})$ - kraftfält



Arbete: $dW = \mathbb{F} \cdot d\mathbb{r} = m \cdot \mathbb{a} \cdot d\mathbb{r}$

$$\frac{dW}{dt} = m\mathbb{a} \cdot \mathbb{v} = m\mathbb{v} \cdot \frac{d\mathbb{v}}{dt} =$$

$$= \frac{1}{2} m \frac{d}{dt}(\mathbb{v} \cdot \mathbb{v}) = \frac{d}{dt}\left(\frac{mv^2}{2}\right)$$

Om $\mathbb{F} = -\nabla \phi$

$$\frac{dW}{dt} = \frac{-\nabla \phi \cdot d\mathbb{r}}{dt} = -\frac{d\phi}{dt}$$

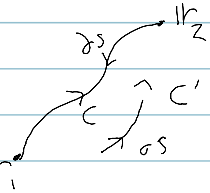
$$\frac{d}{dt}\left(\phi + \frac{mv^2}{2}\right) = 0$$

pot. energi!

Om $\mathbb{F} = -\nabla \phi$

$W = \int_C \mathbb{F} \cdot d\mathbb{r} =$

$= -\int_C \underbrace{\nabla\phi \, d\mathbb{r}}_{d\phi} = -\left(\phi(\mathbb{r}_2) - \phi(\mathbb{r}_1)\right)$



Om $\exists$ potential är W oberoende av vägen $\int_C \mathbb{F} \cdot d\mathbb{r} = \int_{C'} \mathbb{F} \cdot d\mathbb{r}$

$\Updownarrow$

$\oint_{\partial S} \mathbb{F} \cdot d\mathbb{r} = 0$

Stokes:

$\oint_{\partial S} \mathbb{F} \cdot d\mathbb{r} = \int_S \underbrace{\nabla \times \mathbb{F}}_{-\nabla\times\nabla\phi = 0} \, d\mathbb{S} = 0$

$\boxed{\text{Om } \nabla \times \mathbb{F} = 0 \text{ så finns } \phi : \mathbb{F} = -\nabla\phi}$

$\left(\begin{array}{l} \nabla \times \mathbb{F} \text{ rotation} \\ \phi \text{ potential} \end{array} \right)$

ex statiskt el. fält

$\nabla \times \mathbb{E} = 0 \implies \mathbb{E} = -\nabla\phi$

Divergens

$\nabla \cdot \mathbb{F} = \rho \leftarrow$ källa

$\int_{\partial\gamma} \mathbb{F} \cdot d\mathbb{S} = \int_\gamma \rho \, dV$

$\nabla \cdot \nabla \phi = -\rho$

$\Delta \phi = -\rho$

Poissons ekv.

Tvärtom? dvs. $\nabla\cdot\mathbf{G} = 0$, $\nabla\times\mathbf{G} = \mathbf{j} \neq 0$

Kanske $\mathbf{G} = \nabla\times\mathbf{A}$?

Om $\mathbf{G} = \nabla\times\mathbf{A}$: $\nabla\cdot\mathbf{G} = \nabla\cdot(\nabla\times\mathbf{A}) =$

$$= \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) = 0$$

$\phi \longrightarrow \phi + C$ ← konstant

$\mathbf{A} \longrightarrow \mathbf{A} + \nabla\Lambda(\mathbf{r})$ ← skalärfält

Statiskt magnetfält $\mathbf{B}$:

$\nabla\cdot\mathbf{B} = 0$ $\qquad$ $\nabla\times\mathbf{B} = \mathbf{j}$ – ström

$\Longrightarrow \mathbf{B} = \nabla\times\mathbf{A}$, $\underbrace{\nabla\times(\nabla\times\mathbf{A})}_{= -\Delta\mathbf{A} + \nabla(\nabla\cdot\mathbf{A})} = \mathbf{j}$

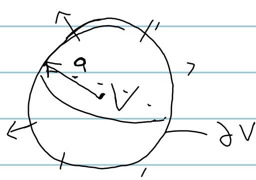
$\boxed{\Delta\mathbf{A} = -\mathbf{j} \text{ om } \nabla\cdot\mathbf{A} = 0}$

Uppgift

Visa att $\nabla(\nabla\cdot\mathbf{F}) - \nabla\times(\nabla\times\mathbf{F}) = \Delta\mathbf{F}$

$$\Delta = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

ex $\mathbf{F} = \dfrac{q}{4\pi}\dfrac{\hat{r}}{r^2}$



$$\int_{\partial V} \mathbf{F}\cdot d\mathbf{S} = \int_{\partial V} \frac{q}{4\pi}\frac{\hat{r}}{a^2}\cdot\hat{r}\, dS =$$

$$= \frac{q}{4\pi a^2}\underbrace{\int_{\partial V} dS}_{4\pi a^2} = q$$

$\nabla\times\mathbf{F} = 0$

$\mathbf{F} = -\nabla\phi$, $\phi = q/\pi 4 r$